# Chain Rule

Eg.

| $x$ | $f(x)$ | $g(x)$ | $f'(x)$ | $g'(x)$ |
|---|---|---|---|---|
| $1/4$ | $2$ | $4/3$ | $16$ | $8/5$ |
| $0$ | $-1$ | $\pi/6$ | $4$ | $1$ |
| $2$ | $0$ | $3$ | $0$ | $5$ |

Say $h(x) = f\left(\sin^2(g(x))\right)$, $k(x) = g(1+[f(x)]^2)$ } Find $h'(0)$ & $k'(0)$.

$$h(x) = \overset{1}{f}\left[\left\{\overset{3}{\sin}(\underset{4}{g(x)})\right\}^{\overset{2}{2}}\right]$$

$$h'(x) = f'\left[\left\{\sin(g(x))\right\}^2\right] \cdot 2\sin(g(x)) \cdot \cos(g(x)) \cdot g'(x)$$

$x=0$

$$h'(0) = f'\left[\left\{\sin(g(0))\right\}^2\right] \cdot 2\sin(g(0)) \cdot \cos(g(0)) \cdot g'(0)$$

$$= f'\left[(\sin \pi/6)^2\right] \cdot 2\sin(\pi/6) \cdot \cos(\pi/6) \cdot 1$$

$$= f'\left[\left(\tfrac{1}{2}\right)^2\right] \cdot 2 \cdot \left(\tfrac{1}{2}\right) \cdot \left(\tfrac{\sqrt{3}}{2}\right) \cdot 1$$

$$= f'\left(\tfrac{1}{4}\right) \cdot \frac{\sqrt{3}}{2}$$

$$= 16 \cdot \frac{\sqrt{3}}{2} = 8\sqrt{3}.$$

$$k(x) = g(1+[f(x)]^2)$$

$$k'(x) = g'(1+[f(x)]^2)\cdot(0+2f(x))\cdot f'(x)$$

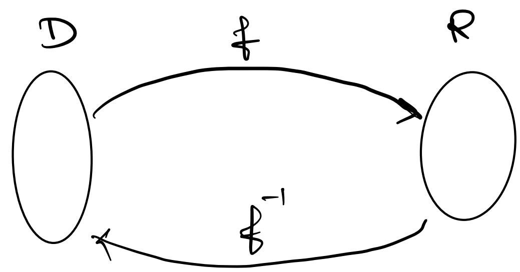
$$k'(0) = g'(1+[f(0)]^2)\cdot(2f(0))\cdot f'(0)$$

$$= g'(1+(-1)^2)\cdot 2(-1)\cdot 4$$

$$= -8\, g'(2)$$

$$= -8(5) = -40$$

<u>Inverse Trigonometric Functions</u>:



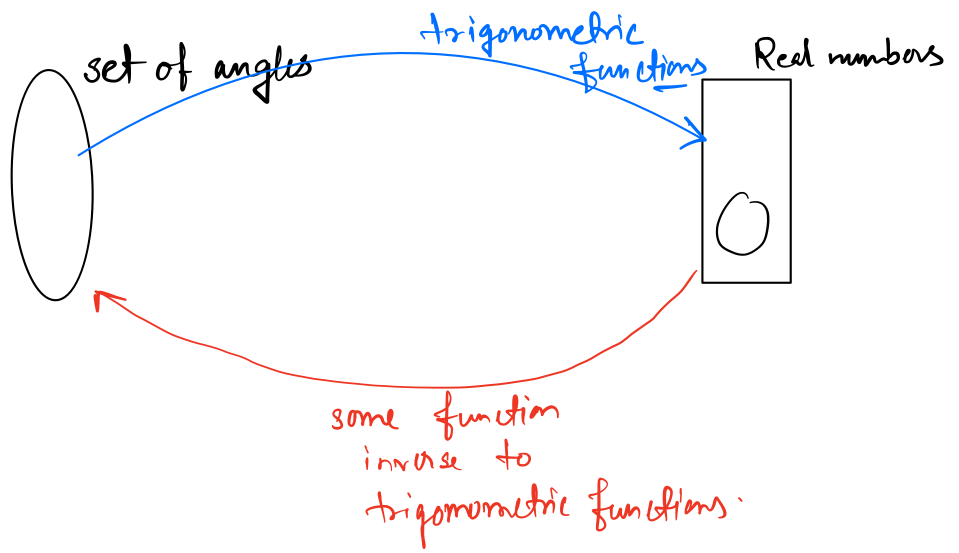

$f(f^{-1}(y)) = y \iff (f\circ f^{-1})(y) = y = id_R(y) \iff f\circ f^{-1} = id_R$

$f^{-1}(f(x)) = x \iff (f^{-1}\circ f)(x) = x = id_D(x) \iff f^{-1}\circ f = id_D$

identity maps –

Say your domain & range both are same as D

$id_D : D \longrightarrow D$ such that $id_D(x) = x$.




$$\left(\sin^{-1}, \quad \cos^{-1}, \quad \tan^{-1}\right)$$
$$\downarrow \qquad \downarrow \qquad \downarrow$$
$$\text{arcsin} \qquad \text{arc cos} \qquad \text{arctan}$$

$$\sin\left(\sin^{-1}(x)\right) = x$$

Find the derivative of $\sin^{-1} x$

$$\sin\left(\sin^{-1}(x)\right) = x$$

Take derivative with respect to $x$ (wrt $x$) on bothside

$$\cos\left(\sin^{-1}(x)\right) \cdot \left[\sin^{-1}(x)\right]' = \frac{d}{dx}(x) = 1.$$

$$\Rightarrow \left[\sin^{-1}(x)\right]' = \frac{1}{\cos\left(\sin^{-1}(x)\right)} = \frac{1}{\sqrt{1-x^2}}$$

$\sin^2\theta + \cos^2\theta = 1$ for any $\theta$

$$\Rightarrow \cos^2\theta = 1 - \sin^2\theta$$

$$\Rightarrow \cos\theta = \sqrt{1-\sin^2\theta}$$ , our $\theta = \sin^{-1}(x)$

$$\cos(\sin^{-1}(x)) = \sqrt{1-[\sin(\sin^{-1}(x))]^2}$$
$$= \sqrt{1-(x)^2}$$
$$= \sqrt{1-x^2}$$

Formulas for Inverse Trigonometric Functions:

when $f(x)=x$

$$\frac{d}{dx}\left[\sin^{-1}(f(x))\right] = \frac{f'(x)}{\sqrt{1-\{f(x)\}^2}} \qquad \frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\cos^{-1}(f(x))\right] = -\frac{f'(x)}{\sqrt{1-\{f(x)\}^2}} \qquad \frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\tan^{-1}(f(x))\right] = \frac{f'(x)}{1+\{f(x)\}^2} \qquad \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left[\cot^{-1}(f(x))\right] = -\frac{f'(x)}{1+\{f(x)\}^2} \qquad \frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}\left[\sec^{-1}(f(x))\right] = \frac{f'(x)}{f(x)\cdot\sqrt{\{f(x)\}^2-1}} \qquad \frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left[\csc^{-1}(f(x))\right] = -\frac{f'(x)}{f(x)\cdot\sqrt{\{f(x)\}^2-1}} \qquad \frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$$